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Three-dimensional model of ray structure formation in cometary plasma tails

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Abstract. The theoretical model of large-scale ray structure formation proposed in a preceding paper (Verkhoglyadova *et al.*, *Lett. Russ. Astro. J.*, **19**, 823–869, 1993) resides in filamentation instability development due to the solar wind flow partially penetrating into the cometary plasma tail. In order to study a real form of the filaments and instability development in space a study of the proposed mechanism in three dimensions is made. Cometary plasma ions and those of the solar wind seem to be collisionless, otherwise collisions of electrons dealing with scattering on ion-acoustic oscillations are characterized by effective collision frequency. The cometary plasma filaments, which are observed as bright rays, are formed due to instability development and take the form of cylindrical plasma fibers with the thickness determined by cometary plasma parameters and those of the solar wind. Under some assumptions we obtain the characteristic length which may be thought of as the maximum ray size. We evaluate the number of rays within one lobe of cometary plasma tail which varies at a range of 2–30 rays. The observational evolution of the ray structure is discussed.

Introduction

The series of bright straight and helical rays with regular spacing are typical for cometary plasma tails. Well-developed ray systems were observed, for example, in plasma tails of comets Morehouse (1908 III), Kohoutek (1973 XII), Bennett (1970 II), Mrkos (1957 III), and P/Halley (1910 II and 1986 III).

Several theoretical models have recently been developed to explain the observed ray structures (see the review in Verkhoglyadova *et al.* (1993)). It is necessary to stress that the paper by Wallis (Wallis, 1967) was the first to study the above problem in all its complexity and to evaluate the ratio length to width of a cometary ray. Although the above-mentioned mechanism may exist in general, it is of interest to consider the more rapid electromagnetic mechanism of ray formation which determines transverse ray size dependence on plasma parameters (Verkhoglyadova *et al.*, 1992, 1993). In the other models known to us the thicknesses of rays do not reach the observed values.

The theoretical model of large-scale stratification studied in a preceding paper (Verkhoglyadova *et al.*, 1993) resides in the consequential development of ion-acoustic and filamentation instabilities due to the solar wind flow penetrating into the cometary plasma. The latter instability consists in division of the solar wind flow and that of cometary plasma into separate current filaments (Miller, 1982) for a one-dimensional case. When the solar wind ion density becomes comparable with that of the cometary plasma the nonlinear instability stage starts and the filaments begin to coalesce. The spatial scale limiting the filament coalescing process is approximately equal to a possible maximum instability wavelength determined from the one-dimensional linear theory (Verkhoglyadova *et al.*, 1993). The above value defines the spatial scale of ray structure and is in good agreement with the observational data.

In order to study a real form of the filaments and instability development in space a study of the proposed mechanism in three dimensions is made.

Model

Interaction between cometary tail plasma, consisting of heavy ions and streamlined protons and electrons of the solar wind flow is studied. Taking into account that the

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cometary magnetosphere consists of two lobes with opposite directions of the magnetic field divided by the current sheet (Tsurutani, 1991), one undertakes the study within one lobe, i.e. in cometosheath with thickness of $L = 5\times 10^4$–10^5 km which sufficiently exceeds the characteristic thickness of the rays. Therefore the system is considered to be infinite. If the magnetic field direction becomes just the opposite, this does not change the obtained results. The system is assumed to be homogeneous. The flow velocity of cometary ions and electrons is neglected ($V_{0i2} = V_{0e2} = 0$) in comparison with the flow velocity of that of the solar wind $V_{0e1} = V_{0i1} = 5\times 10^5$ m s^{-1} (Bame *et al.*, 1986). The indices 1 and 2 are referred to the solar wind and comet components, respectively, index 'e' denotes the electron component and 'i' denotes the ion one. Densities of the solar wind electrons (n_{e1}) and ions ($n_{i1} = n_{e1}$) are assumed to be equal to 5×10^{-6} m^{-3}, $n_{e2} = n_{i2} = 50\ n_{i1}$ (Bame *et al.*, 1986; McComas *et al.*, 1987). The magnetic field in the solar wind and cometary plasma interaction region is considered to be $H_0 = 10$ nT (McComas *et al.*, 1987; Perez de-Tejada, 1990; Tsurutani, 1991) and is directed approximately along the solar wind flow. The typical plasma parameters are $\omega_{He} \simeq 2\times 10^3\ s^{-1}$, $\omega_{pe1} \simeq 10^5\ s^{-1}$, $\omega_{Hi1} \simeq 1\ s^{-1}$, $\omega_{pi1} \simeq 3\times 10^3\ s^{-1}$. We use the following designations, namely m_α are the masses, $\omega_{H\alpha} = eH_0/(m_\alpha c)$ and $\omega_{p\alpha} = (4\pi e^2 n_\alpha/m_\alpha)^{1/2}$ are the corresponding cyclotron frequencies and plasma frequencies for particles of type $\alpha = e_1, e_2, i_1, i_2$.

It is common knowledge that ion-acoustic waves exist in a cometary plasma (Scarf *et al.*, 1986; Tsurutani, 1991). As we have noted previously (Verkhoglyadova *et al.*, 1993), the ion-acoustic oscillations could result from electron flow instability of the solar wind. Ion-acoustic turbulence development in plasma is responsible for effective scattering of cometary plasma electrons and those of the solar wind on the above-mentioned oscillations (Akhieser *et al.*, 1975). In the problem under consideration we can describe the above effect in terms of effective frequency of scattering of electrons (ν_{eff}). It should be noted that the origin of the dissipation mechanism for electrons does not affect the proposed model. Classical collisions, if they are realized in the cometary plasma under the certain conditions, can be taken into account by the same procedure.

Linear theory of filamentation instability of the solar wind in cometary tail plasma

We will consider the filamentation instability of the solar wind penetrating into the cometary tail plasma where ion-acoustic turbulence has been formed. A study will be made in terms of hydrodynamics equations for ions and electrons (Akhieser *et al.*, 1975). Physical processes with spatial scales exceeding the gyro radii of electrons and ions and $\omega_{p\alpha} > \omega_{H\alpha}$ are investigated. As is evident from the previous paper (Verkhoglyadova *et al.*, 1993), the minimum possible instability wavelength is determined by thermal straggling of the particles. This fact may be explained quantitatively if one takes into account that straggling of the velocities diffuses the density disturbance if the wavelength becomes equal to the mean particle path. Our main interest here is with greater spatial scales which are not affected by thermal effects. Consequently, all the components are considered to be cold. Cometary plasma ions and those of the solar wind seem to be collisionless, otherwise collisions of electrons dealing with either scattering on ion-acoustic oscillations or another dissipation mechanism will be characterized by effective collision frequency ν_{eff} (Verkhoglyadova *et al.*, 1993). The system of cylindrical coordinates with Z axis coinciding with the tail axis is introduced ($H_0 \parallel Z$). Considering small disturbances of physical values proportional to $\exp(i\omega t)$ in the framework of the system of equations of motion for the electron and ion components, after linearization and simple transformations we will obtain the following equations for electrical field components:

$$\chi E_r + i\lambda E_\varphi - \frac{ic\eta}{\omega}\frac{\partial E_z}{\partial r} = 0,$$

$$\chi E_\varphi + \frac{c^2}{\omega^2}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{\partial r} rE_\varphi + \frac{c\theta}{\omega}\frac{\partial E_z}{\partial r} = 0,$$

$$\rho E_z - \frac{ic\eta}{\omega}\frac{1}{r}\frac{\partial}{\partial r} rE_r - \frac{c\theta}{\omega}\frac{1}{r}\frac{\partial}{\partial r} rE_\varphi + \frac{c^2}{\omega^2}(1+\mu)\frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial E_z}{\partial r} = 0, \quad (1)$$

where $\chi, \lambda, \eta, \theta, \rho, \mu$ are the following functions on plasma parameters:

$$\chi = 1 - \sum_\alpha \sigma_\alpha \frac{\xi_\alpha}{\omega}, \quad \lambda = \sum_\alpha \sigma_\alpha \frac{\omega_{H\alpha}}{\omega}, \quad \eta = \sum_\alpha \sigma_\alpha \frac{V_{0\alpha}}{c}\frac{\xi_\alpha}{\omega},$$

$$\theta = \sum_\alpha \sigma_\alpha \frac{V_{0\alpha}}{c}\frac{\omega_{H\alpha}}{\omega}, \quad \rho = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\xi_\alpha \omega}, \quad \mu = \sum_\alpha \sigma_\alpha \frac{V_{0\alpha}^2}{c^2}\frac{\xi_\alpha}{\omega},$$

$$\sigma_\alpha = \frac{\omega_{p\alpha}^2}{(\xi_\alpha^2 - \omega_{H\alpha}^2)}, \quad \xi_\alpha = \omega - i\nu_\alpha. \quad (2)$$

Here $\nu_\alpha = \nu_{eff}$ is assumed for $\alpha = e_1, e_2$ and $\nu_\alpha = 0$ for $\alpha = i_1, i_2$; $V_{0\alpha} \neq 0$ for $\alpha = e_1, i_1$ and $V_{0\alpha} = 0$ for $\alpha = e_2, i_2$.

Assuming the cylindrical symmetry of the flow and of the cometary tail it would appear reasonable to find solution of the system (1) in the following form:

$$E_z = E_z^0 \mathscr{T}_0(kr), \quad E_r = E_r^0 \mathscr{T}_1(kr), \quad E_\varphi = E_\varphi^0 \mathscr{T}_1(kr), \quad (3)$$

where $\mathscr{T}_0$ and $\mathscr{T}_1$ are the zero-power and first-power Bessel functions and k plays the role of a transverse wave vector. We neglect the other part of solution (3) because of the requirement of its finite value in the point of $r = 0$.

Substituting (3) into (1) and using the properties of Bessel functions (Abramowitz and Stegun, 1964) we obtain the following equation for k:

$$\rho - \frac{c^2k^2}{\omega^2}\left[1 + \mu + \frac{\eta^2}{\chi} + \theta\frac{\theta + \dfrac{\eta\lambda}{\chi}}{\chi - \dfrac{c^2k^2}{\omega^2}}\right] = 0. \quad (4)$$

We perform an analytical analysis assuming in (4) that $\nu_{\text{eff}} \neq 0$, $\omega_{\text{pe2}} \gg \omega_{\text{pe1}}$, $\omega_{\text{pi2}} \gg \omega_{\text{pi1}}$. For small k region, $|\omega| \ll \nu_{\text{eff}}$, $|\omega|_{\text{Hi1}}$ and $\nu_{\text{eff}} \leqslant \omega_{\text{He}}$ we obtain:

$$-\frac{\omega_{\text{pi2}}^2}{\omega^2}+\frac{\omega_{\text{pc}}^2}{i\nu_{\text{eff}}\omega}+\frac{k^2V_{0\text{i}1}^2\omega_{\text{pi1}}^2}{\omega^2\omega_{\text{Hi1}}^2} -\frac{k^2V_{0\text{e}1}^2\omega_{\text{pe1}}^2}{\omega^2\omega_{\text{He}}^2}\frac{i\nu_{\text{eff}}}{\omega}-\frac{k^2c^2}{\omega^2}=0.$$

The form of equation coincides with that of the dispersion equation for filamentation instability in a one-dimensional case (Verkhoglyadova *et al.*, 1993). Further, only the aperiodic solutions with $\text{Re}\,\omega = 0$ will be considered. The filamentation instability occurs under the condition of $\text{Im}\,\omega < 0$. The exponentially exceeding in time and quasi-periodic in space disturbances of physical values, namely the plasma densities, magnetic field, etc., correspond to the above case. The filamentation instability is physically concerned with attracting (pinching) of currents streaming in the same direction (Miller, 1982). One notes the existence of the same wave number value as in Verkhoglyadova *et al.* (1992, 1993):

$$k_0 = \frac{\omega_{\text{pi2}}\omega_{\text{Hi1}}}{V_{0\text{i}1}\omega_{\text{pi1}}}, \tag{5}$$

and k_0 may be considered as the minimum wave number. Using the properties of Bessel functions (Abramowitz and Stegun, 1964) we can estimate the possible ray thickness as $l_{\max} = 2.4/k_0 \simeq 10^3$–$10^4$ km. Setting the region occupied by one ray as the distance between the point of $r = 0$ and the second zero of $\mathcal{T}(k_0 r)$ function it is easy to evaluate the number of rays within one lobe of the cometary plasma tail which is approximately equal to $N \simeq L \times k_0/5.6$ and varies at a range of 2–30 rays.

Possible observational evolution of the ray structure

The cometary plasma density and that of the solar wind, magnetic field and ion composition exert a primary control over a maximum ray thickness $l_{\max}$ (5). So, we can trace possible evolution of the ray structure during comet motion within the Solar System.

As far as cometary ion density varies with the distance from the Sun approximately as $n_{\text{i2}} = n_{0\text{i}2}\cdot r^{-3.3}$ (Churyumov *et al.*, 1990) and the proton density of the solar wind as $n_{\text{i1}} = n_{0\text{i}1}\cdot r^{-2}$ where $n_{0\text{i}1}$ and $n_{0\text{i}2}$ are those of the solar wind and the cometary plasma at the heliocentric distance of 1 A.U., then $l_{\max}(r) \simeq l_{\max}(r = 1\ \text{A.U.})\cdot r^{0.65}$.

The minimum ray thickness is obtained near the Sun and runs to $0.6\cdot l_{\max}(r = 1\ \text{A.U.})$ for 0.46 A.U. The thickness ranges up to $1.8\cdot l_{\max}(r = 1\ \text{A.U.})$ approximately at 2.5 A.U., i.e. at the distance where the condition $\omega_{\text{pi1}} \simeq \omega_{\text{pi2}}$ is satisfied. At greater distances from the Sun the reverse situation of high density flow instability in less dense plasma is realized. The study made by Verkhoglyadova *et al.* (1993) remains applicable, while the indices 1 and 2 need to be exchanged. Thereby we obtain the following dependency: $l_{\max}(r) \simeq l_{\max}(r = 1\ \text{A.U.})\cdot r^{-0.65}$ and the estimation for ray thickness as $0.24\cdot l_{\max}(r = 1\ \text{A.U.})$ for heliocentric distance of 9 A.U.

Maximum ray thickness decreases gradually with moving away from the region where $\omega_{\text{pi1}} \simeq \omega_{\text{pi2}}$. Consequently, creation of thinner rays can be observed against the background of big diffuse ones. Generally, the above effect looks like splitting of the ray. It is useful to bear in mind that the ray thickness slightly varies with the comet motion, approximately by two times, in terms of our model.

The above estimations are suitable for dissipative regime of the filamentation instability, i.e. if ($\nu_{\text{eff}} \neq 0$). The ion acoustic instability threshold depends exponentially on plasma temperature. Thereby its increase causes the ion acoustic instability stabilization. One would expect a filamentation instability transition to the nondissipative regime as the Sun is approached. In this case the cometary ray thickness is decreased by an order of magnitude. This effect would be observed as a sudden disappearance of the big rays. The above process would go on in a different way provided the system dissipation is caused by development of other instabilities being less effective than the ion-acoustic one at the ordinary conditions.

This consideration is applicable for the quasistationary process, namely when ray structure formation time exceeds sufficiently the typical change time of the plasma parameters during the comet motion. According to the 1D-computer simulation results (Verkhoglyadova *et al.*, 1993) the ray structure evolution in the nonlinear regime takes place in the time interval of $60/\gamma$, where γ is the growth rate of the filamentation instability. For the typical plasma parameters it comprises several seconds and the process may be thought of as a quasistationary one. In the above case the magnetic field changes of the solar wind are negligible. It should be noted that $l_{\max}$ is proportional to $m_{\text{i2}}^{0.5}$, i.e. the ray thickness is dissimilar for distinct ion composition of the cometary plasma tails.

An explanation of frequently observed motion of the rays inside to the tail axis, i.e. the phenomenon of the 'closing-in' rays, arouses considerable interest in the context of our model. We assume the existence of weak plasma flow with $V_\perp$ into the cometary plasma tail. It is common knowledge that the cometary ionosphere is created as the Sun is approached. At first a neutral and then an ionized tail is penetrated by the solar wind. Under certain conditions the ray structure is formed, i.e. a system of alternating sequences of rapid flows of the solar wind origin and slow flows of the cometary plasma. According to our model the above picture would be observed inside the tail far apart from the extended transition region. The supposition is not pertinent to formation of plasma boundaries around the cometary head, because the above processes touch upon the regions far from the head and relates only to the plasma tail structure. The evidence invoked in favor of our proposal are the data obtained by ICE during its flight through the Giacobini–Zinner tail. The fluctuations of antisunward plasma flow velocities were measured against a background of smooth decrease of velocity toward the tail axis (McComas *et al.*, 1987). The velocity gradient in the neighboring plasma regions was registered to be around 200 km s^{-1}, i.e. comparable with the solar wind velocity rather than with that of cometary plasma. Typical scale of the above fluctuations is about 10^3 km across the tail, which is a comparable-size with the cometary rays or streamers. In parallel with those facts the analogous behavior of infrozen magnetic field components was detected (Smith *et al.*, 1986; Huebner *et al.*, 1989). In McComas *et al.*'s paper (McComas *et al.*, 1987), it was assumed that the solar wind flow and magnetic field had access to much larger cometary regions than the transition one. This assumption does not contradict theoretical models of the cometary magnetosphere in the region of the plasma tail, for instance, with the one proposed in Malara *et al.*'s paper (Malara *et al.*, 1989). It was suggested that the plasma flow towards the current sheet increased away from the nuclei. The small transverse velocity results in a small real part of the growth rate ($\mathrm{Re}\,\omega \neq 0$) and, respectively, in small temporal density variations in a fixed point of the cometary tail approximately with the period of $l_{max}/V_{\perp}$. One can conclude that the wave of stratification becomes the travelling one, so the ray structure collapses to the axis with the mean velocity of $V_{\perp}$. Perhaps, this effect causes the 'closing-in' of the rays.

Let us consider a possible influence of plasma inhomogeneity assuming a smooth increase of plasma density and slowing-down of the plasma flow inside the tail. The respective velocity and plasma density changes are negligible and do not affect the filamentation instability development in the region under consideration of the sizes about 10^4–10^5 km and far from the current sheet (McComas *et al.*, 1987; Tsurutani, 1991).

Conclusion

According to our consideration the large-scale ray structure is resulted from filamentation instability development in cometary tail plasma with well-developed ion-acoustic turbulence. The above effect results in the solar wind flow division into current filaments. Owing to plasma quasineutrality, spaces between the above filaments are filled with cometary plasma ones, which are observed as bright rays. They take the form of cylindrical plasma fibers (3). Their thicknesses are determined by cometary plasma parameters and those of the solar wind (5) and reach the values of 10^3–10^4 km. The above values increase with an increase in the solar wind velocity and density. The obtained spatial scales are in good agreement both with one-dimensional theory (Verkhoglyadova *et al.*, 1993) and the observational data. The total number of cometary rays is of the order of 2–30.

We would like to outline briefly the main features of the observational evolution of the cometary ray structure as they are evident from the paper. The proposed model would explain the general features of the ray structure formation during comet motion which consists, in consequent creation, of more and more sharp rays, their quick coalescing and slow splitting, possible quick disappearance as the Sun is approached. Our consideration is based on assumptions of a high level of wave activity in tail plasma and partial penetrating of the solar wind into cometary tail. This point of view is caused by the ICE data and picture of interaction of the solar wind and an initially neutral coma. Weak plasma flow into the tail is also suggested. The latter effect tends to the phenomenon of 'closing-in' rays.

It should be noted that the above study was made in terms of infinite homogeneous cometary plasma. The implementation of the proposed mechanism to inhomogeneous plasma case is a subject of further investigation.

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