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Shrinkage estimation in exponential type-II censored data under LINEX loss

Gyan Prakash*, D.C. Singh

Department of Statistics, Harishchandra P. G. College, Varanasi, India

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Abstract

This paper deals with the study of the performance of the shrinkage testimators under the invariant version of LINEX loss function for the scale parameter of an exponential distribution when type-II censored data are available.

1. Introduction

In life-testing research, the most widely used life distribution is the exponential with probability density function for any random variable x;

$$f(x,\theta)=\begin{cases}\dfrac{1}{\theta}\mathrm{e}^{-\frac{x}{\theta}}; & x\geq 0,\theta\geq 0\\ 0; & \text{otherwise.}\end{cases}$$

Let $x_1, x_2, \text{- - -}, x_n$ be the random samples of size n taken form the exponential distribution. The parameter θ is called the scale parameter, better known as average life. The mean time to failure is θ and $\overline{x}$ is the unbiased estimate for θ. Further, the survival function is $S(x) = \mathrm{e}^{-\theta x}$ and the failure rate $\gamma = \frac{1}{\theta}$. When the failure rate is constant, the exponential model has been found to be useful. When failure rate varies with the time, the Weibull model is available. In the present paper, concentration is on the exponential model.

In the estimation of the average life or the reliability function, use of symmetric loss function (squared error) may be inappropriate (Basu & Ebrahimi, 1991). The squared error loss has been considered as equal weightage to the positive and negative errors for the estimation. Zellner (1986) proposed an asymmetric loss function generally known as the LINEX (linear-exponential) loss function. Following Basu and Ebrahimi (1991), the invariant form of the LINEX loss function (ILL) for any parameter θ is defined as

$$L(\Delta)=\left\{\mathrm{e}^{c\Delta}-c\Delta-1\right\};\quad c\neq 0,\ \Delta=\left(\frac{\hat{\theta}}{\theta}-1\right), \tag{1.1}$$

where c is the shape parameter and $\hat{\theta}$ is any estimate of the parameter θ.

* Corresponding author.
E-mail addresses: ggyanji@yahoo.com (G. Prakash), dc_singh52@rediff.com (D.C. Singh).

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The LINEX loss function is convex and the shape of this loss function is determined by the value of c. The negative (positive) value of c gives more weight to overestimation (underestimation) and its magnitude reflects the degree of asymmetry. It is seen that, for $c = 1$, the function is quite asymmetric with overestimation being costlier than underestimation. If $c < 0$, it rises almost exponentially when the estimation error $\left(\hat{\theta} - \theta\right) < 0$ and almost linearly when $\left(\hat{\theta} - \theta\right) > 0$. For small values of $|c|$, the LINEX loss function is almost symmetric and not far from squared error loss function.

Pandey (1997), Parsian and Farsipour (1999), Singh, Gupta, and Upadhyay (2002), Misra and Meulen (2003), Ahmadi, Doostparast, and Parsian (2005), Xiao, Takada, and Shi (2005), Singh, Prakash, and Singh (2007) and others have used the LINEX loss function in the various estimation and prediction problems.

Searls (1964) found a minimum mean square error estimator for the parameter θ in the exponential distribution as $\frac{n}{n+1}\bar{x}$ under the class of $l\bar{x}$. Pandey (1997) showed that this estimator was inadmissible under the LINEX loss function.

In life-testing, fatigue failures and other kinds of destructive test situations, the observations usually occurred in an ordered manner such a way that the weakest items failed first and then the second one and so on. Epstein and Sobel (1953) proved that the total test time

$$T_r = \sum_{i=1}^{r} x_{(i)} + (n-r)x_{(r)}; \quad n > r \tag{1.2}$$

is the complete sufficient statistic for θ, where r is the uncensored sample size. The minimum variance unbiased estimator for the parameter θ is $\frac{T_r}{r}$.

Pandey, Srivastava, and Mishra (2004) considered a class for the total test time as $Y = d\frac{T_r}{r}$, and found the value of the constant $d = \frac{r}{c}\left(1 - \mathrm{e}^{-\frac{c}{r+1}}\right) = d_1$ (say) which minimizes the risk of Y under the ILL. The minimum risk estimator is $Y_1 = d_1\frac{T_r}{r}$ with the minimum risk under ILL

$$\mathrm{Risk}(Y_1) = c + (r+1)\left(\mathrm{e}^{-\frac{c}{r+1}} - 1\right). \tag{1.3}$$

In the present paper, some shrinkage testimators are proposed for the scale parameter of an exponential distribution when type-II censored data are available and the performance of these testimators with respect to improved estimator under the ILL have been studied.

2. Shrinkage testimators and their properties

Following Thompson (1968), the shrinkage estimator for the parameter θ is given by

$$\hat{Y} = \theta_0 + k\left(\frac{T_r}{r} - \theta_0\right), \quad 0 \le k \le 1. \tag{2.1}$$

The value of the shrinkage factor k near to the zero implies strong belief in the guess value θ_0 and near to one implies a strong belief in the sample values. Several researchers have studied the performance of the shrinkage estimators and found that the shrinkage estimator performs better with respect to any usual estimator when the guess value θ_0 is close to the parameter θ. This suggests that we may test the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. A test statistic $\frac{2T_r}{\theta_0} \backsim \chi^2_{(2r)}$ is available for testing the hypothesis H_0.

The loss for estimator $\hat{Y}$ under the ILL is defined as

$$L(\hat{Y}) = \left\{\mathrm{e}^{c\Delta} - c\Delta - 1\right\},$$

where $c\Delta = c\left\{\delta - 1 + k\left(\frac{T_r}{r\theta} - \delta\right)\right\}$ and $\delta = \frac{\theta_0}{\theta}$.

The risk of the proposed shrinkage estimator $\hat{Y}$ under the ILL is given by

$$\mathrm{Risk}(\hat{Y}) = \mathrm{e}^{c(\delta-1)}\mathrm{e}^{-ck\delta}\left(1 - \frac{ck}{r}\right)^{-r} - c(\delta-1)(1-k) - 1. \tag{2.2}$$

The value of $k = k_{\min}$ (say), which minimizes the risk $\text{Risk}(\hat{Y})$ is thus obtained by solving the given equation

$$(1-\delta)\mathrm{e}^{c(1-\delta)} = \mathrm{e}^{-ck\delta}\left(1-\frac{ck}{r}\right)^{-(r+1)}\left\{1-\delta\left(1-\frac{ck}{r}\right)\right\}. \tag{2.3}$$

The value of $k_{\min}$ depends upon the unknown parameter θ. Hence, an estimate $\hat{k}$ of $k_{\min}$ is obtained by replacing the parameter θ to its minimum variance unbiased estimator. Based on this, the proposed shrinkage testimator for the scale parameter θ is defined as

$$\hat{\theta}_1 = d_1\frac{T_r}{r} + \left(\left(1-\hat{k}\right)\theta_0 + \left(\hat{k}-d_1\right)\frac{T_r}{r}\right) I_{(t_1 \le T_r \le t_2)}, \tag{2.4}$$

where $I_{(A)}$ denotes the indicator of A, $t_1 = \frac{m_1\theta_0}{2}$ and $t_2 = \frac{m_2\theta_0}{2}$. Here m_1 and m_2 are the values of the lower and upper $100\frac{\alpha}{2}\%$ points of the chi-square distribution with $2r$ degrees of freedom. The risk under the ILL for the shrinkage testimator $\hat{\theta}_1$ is given by

$$\begin{aligned}\text{Risk}\left(\hat{\theta}_1\right) = &\left\{\mathrm{e}^{c(\delta-1)}G\left(w_1, w_2, \mathrm{e}^{f}\right) - \mathrm{e}^{-c}G\left(w_1, w_2, f_0\right) - G\left(w_1, w_2, f\right) - c\delta G\left(w_1, w_2, 1\right)\right. \\ &\left. +\ G\left(w_1, w_2, f_0\right) + c + (1+r)\left(\exp\left(-\frac{c}{r+1}\right) - 1\right)\right\},\end{aligned} \tag{2.5}$$

where $w_1 = \delta\frac{m_1}{2}$, $w_2 = \delta\frac{m_2}{2}$, $f = c\hat{k}\left(\frac{w}{r}-\delta\right)$, $f_0 = \left(w\frac{cd_1}{r}\right)$, $G(u,v,y) = \frac{1}{\Gamma(r)}\int_u^v (y).\mathrm{e}^{-w}w^{r-1}\mathrm{d}w$ and y may be a function of w.

Waikar, Schuurmann, and Raghunathan (1984) has suggested an idea of selecting the shrinkage factor which is the function of the test statistic i.e., under $H_0 : \theta = \theta_0$

$$m_1 \le \frac{2T_r}{\theta_0} \le m_2 \Leftrightarrow 0 \le k_1 \text{ (say)} = \frac{1}{m_2 - m_1}\left(\frac{2T_r}{\theta_0} - m_1\right) \le 1.$$

Therefore, the proposed shrinkage testimator based on k_1 is given by

$$\hat{\theta}_2 = d_1\frac{T_r}{r} + \left(\left(1-k_1\right)\theta_0 + \left(k_1-d_1\right)\frac{T_r}{r}\right) I_{(t_1 \le T_r \le t_2)}. \tag{2.6}$$

The risk under the ILL for the shrinkage testimator $\hat{\theta}_2$ is given by

$$\begin{aligned}\text{Risk}\left(\hat{\theta}_2\right) = &\left\{\mathrm{e}^{c(\delta-1)}G\left(w_1, w_2, \mathrm{e}^{f_1}\right) - \mathrm{e}^{-c}G\left(w_1, w_2, f_0\right) - G\left(w_1, w_2, f_1\right) - c\delta G\left(w_1, w_2, 1\right)\right. \\ &\left. +\ G\left(w_1, w_2, f_0\right) + c + (1+r)\left(\exp\left(-\frac{c}{r+1}\right) - 1\right)\right\},\end{aligned} \tag{2.7}$$

where $f_1 = \frac{c}{m_2-m_1}\left(\frac{2w}{\delta} - m_1\right)\left(\frac{w}{r} - \delta\right)$.

When $H_0 : \theta = \theta_0$ is accepted, $m_1 \le 2r \le m_2 \Rightarrow \frac{m_1}{2r} \le 1$. If one is interested in taking smaller values of the shrinkage factor, he can take $\frac{m_1}{2r} \cong 1$. The proposed shrinkage testimator is

$$\hat{\theta}_3 = d_1\frac{T_r}{r} + \left(\left(1-k_2\right)\theta_0 + \left(k_2-d_1\right)\frac{T_r}{r}\right) I_{(t_1 \le T_r \le t_2)}, \tag{2.8}$$

where $k_2 = \frac{2r}{m_2-m_1}\left|\frac{T_r}{r\theta_0} - 1\right|$; it may be possible that the value of shrinkage factor is negative so positive is taken. Adke, Waikar, and Schuurmann (1987) and Pandey, Malik, and Srivastava (1988) have considered this type of shrinkage factor. The risk of the shrinkage testimator $\hat{\theta}_3$ is given by

Table 1
$RE\left(\hat{\theta}_1, Y_1\right)$

a	α	δ							
		0.04	0.06	0.80	1.00	1.20	1.40	1.60	1.80
$r = 06$									
	0.01	1.0711	1.7877	2.7581	5.4036	3.2634	2.2224	1.6493	1.4161
0.25	0.05	1.0427	1.7504	2.7375	4.7985	3.3633	2.3124	1.7592	1.4828
	0.10	1.0290	1.6824	2.7046	4.7222	3.5432	2.5596	1.9372	1.5897
	0.01	1.0751	1.8178	2.8254	5.4982	3.2897	2.1130	1.5498	1.3231
0.50	0.05	1.0436	1.7619	2.7855	4.9775	3.3926	2.2354	1.6799	1.4015
	0.10	1.0293	1.6861	2.7205	4.8654	3.6183	2.5108	1.8806	1.5236
	0.01	1.0820	1.8755	2.9654	5.7395	3.3335	1.9106	1.3590	1.1429
1.00	0.05	1.0447	1.7768	2.9664	5.3670	3.4128	2.0986	1.5245	1.2462
	0.10	1.0293	1.6871	2.7363	5.1446	3.7096	2.4282	1.7639	1.3980
	0.01	1.0875	1.9256	3.1112	6.0564	3.4024	1.7257	1.1804	1.0971
1.50	0.05	1.0449	1.7858	3.0284	5.7925	3.4442	1.9676	1.3727	1.0976
	0.10	1.0288	1.6850	2.7356	5.4030	3.8007	2.3395	1.6399	1.2720
$r = 08$									
	0.01	1.0337	1.6667	2.4681	4.7034	3.2548	2.0570	1.5964	1.4099
0.25	0.05	1.0161	1.6053	2.4521	4.4303	3.3430	2.2262	1.7188	1.4722
	0.10	1.0097	1.5505	2.3125	4.2541	3.4158	2.5243	1.9026	1.5699
	0.01	1.0352	1.6857	2.5233	4.7991	3.2077	1.9690	1.5160	1.3349
0.50	0.05	1.0165	1.6112	2.5087	4.5659	3.2785	2.1681	1.6584	1.4079
	0.10	1.0099	1.5524	2.3171	4.3378	3.4887	2.4953	1.8642	1.5195
	0.01	1.0379	1.7205	2.6317	5.0251	3.0918	1.8056	1.3596	1.1878
1.00	0.05	1.0168	1.6176	2.5467	4.8480	3.1981	2.0695	1.5389	1.2842
	0.10	1.0098	1.5521	2.3144	4.4912	3.5631	2.4564	1.7860	1.4240
	0.01	1.0395	1.7494	2.7391	5.3007	2.9919	1.6526	1.2103	1.0448
1.50	0.05	1.0168	1.6208	2.5724	5.1387	3.2239	1.9723	1.4238	1.1651
	0.10	1.0097	1.5503	2.3040	4.6211	3.6341	2.4111	1.7084	1.3316
$r = 10$									
	0.01	1.0152	1.5848	2.2875	4.3053	2.9524	1.9581	1.5642	1.4066
0.25	0.05	1.0059	1.5191	2.2216	4.1720	3.0568	2.1895	1.6962	1.4660
	0.10	1.0032	1.4790	2.0447	3.8621	3.3903	2.5240	1.8836	1.5575
	0.01	1.0158	1.5965	2.3315	4.4001	2.9226	1.8854	1.4970	1.3441
0.50	0.05	1.0060	1.5222	2.2354	4.2722	3.0993	2.1469	1.6480	1.4132
	0.10	1.0032	1.4800	2.0446	3.9059	3.4631	2.5131	1.8560	1.5174
	0.01	1.0168	1.6167	2.4140	4.6147	2.8334	1.7494	1.3657	1.2203
1.00	0.05	1.0061	1.5247	2.2516	4.4721	3.1243	2.0760	1.5541	1.3108
	0.10	1.0032	1.4794	2.0368	3.9800	3.5318	2.5056	1.8027	1.4405
	0.01	1.0173	1.6327	2.4914	4.8641	2.7550	1.6216	1.2389	1.0990
1.50	0.05	1.0061	1.5257	2.2590	4.6666	3.1525	2.0098	1.4636	1.2121
	0.10	1.0031	1.4781	2.0245	4.0346	3.5948	2.5003	1.7515	1.3668

$$\text{Risk}\left(\hat{\theta_3}\right) = \left\{ e^{c(\delta-1)} G\left(w_1, w_2, e^{f_2}\right) - e^{-c} G\left(w_1, w_2, f_0\right) - G\left(w_1, w_2, f_2\right) - c\delta G\left(w_1, w_2, 1\right) + G\left(w_1, w_2, f_0\right) + c + (1+r)\left(\exp\left(-\frac{c}{r+1}\right) - 1\right) \right\}, \tag{2.9}$$

where $f_2 = \frac{c}{m_2 - m_1}\left|\frac{2w}{\delta} - 2r\right|\left(\frac{w}{r} - \delta\right)$.

The minimum value of constant d, d_1 obtained for the class $Y = d\frac{T_r}{r}$, lies between zero and one. Hence, it may be a choice for the shrinkage factor. Thus, the proposed shrinkage testimator may be considered as

Table 2
$RE\left(\hat{\theta}_2, Y_1\right)$

a	α	δ							
		0.04	0.06	0.80	1.00	1.20	1.40	1.60	1.80
$r = 06$									
	0.01	1.1044	2.5649	3.4315	3.9436	3.2962	2.0350	1.1377	0.6578
0.25	0.05	1.0499	2.2938	3.2981	4.7258	4.4923	2.6305	1.3741	0.7620
	0.10	1.0310	2.1435	2.9690	4.6073	5.4137	3.3163	1.6224	0.8523
	0.01	1.1087	2.6103	3.5124	3.9748	3.1634	1.9266	1.0715	0.6143
0.50	0.05	1.0502	2.3040	3.3525	4.8562	4.5185	2.5590	1.3105	0.7163
	0.10	1.0308	2.1448	2.9896	4.7157	5.5703	3.3001	1.5674	0.8070
	0.01	1.1151	2.6932	3.6824	4.0927	2.9082	1.7187	0.9451	0.5326
1.00	0.05	1.0503	2.3185	3.4507	5.1308	4.5799	2.4204	1.1890	0.6303
	0.10	1.0301	2.1445	3.0201	4.9235	5.9086	3.2689	1.4607	0.7211
	0.01	1.1191	2.7645	3.8598	4.2634	2.6645	1.5222	0.8264	0.4575
1.50	0.05	1.0499	2.3262	3.5330	5.4171	4.6472	2.2856	1.0748	0.5513
	0.10	1.0293	2.1410	3.0375	5.1122	6.2736	3.2360	1.3579	0.6413
$r = 08$									
	0.01	1.0453	2.2927	3.1444	3.9246	3.4155	2.0149	1.0569	0.5826
0.25	0.05	1.0179	2.0694	2.8298	4.2777	4.6307	2.6856	1.2996	0.6775
	0.10	1.0101	1.9740	2.5530	3.9464	5.2809	3.3931	1.5345	0.7534
	0.01	1.0468	2.3179	3.2154	3.9522	3.3215	1.9280	1.0042	0.5481
0.50	0.05	1.0179	2.0736	2.8592	4.3798	4.6933	2.6344	1.2484	0.6411
	0.10	1.0100	1.9741	2.5618	4.0091	5.4303	3.3847	1.4899	0.7168
	0.01	1.0490	2.3619	3.3597	4.0363	3.1404	1.7605	0.9033	0.4831
1.00	0.05	1.0179	2.0792	2.9099	4.5881	4.8328	2.5354	1.1503	0.5721
	0.10	1.0097	1.9730	2.5736	4.1241	5.7492	3.3807	1.4030	0.6470
	0.01	1.0503	2.3976	3.5049	4.1904	2.9665	1.6011	0.8081	0.4229
1.50	0.05	1.0176	2.0814	2.9497	4.7975	4.9882	2.4395	1.0574	0.5080
	0.10	1.0094	1.9706	2.5785	4.2228	6.0903	3.3798	1.3190	0.5816
$r = 10$									
	0.01	1.0191	2.1143	2.8872	3.8568	3.5285	2.0040	0.9928	0.5251
0.25	0.05	1.0063	1.9476	2.5199	3.8679	4.7058	2.7483	1.2398	0.6119
	0.10	1.0032	1.8879	2.2989	3.4602	5.0848	3.5645	1.4680	0.6780
	0.01	1.0196	2.1281	2.9419	3.9088	3.4635	1.9313	0.9487	0.4964
0.50	0.05	1.0063	1.9494	2.5361	3.9419	4.7892	2.7114	1.1967	0.5814
	0.10	1.0032	1.8877	2.3026	3.4965	5.2153	3.5486	1.4300	0.6471
	0.01	1.0204	2.1513	3.0495	4.0230	3.3387	1.7905	0.8641	0.4420
1.00	0.05	1.0062	1.9513	2.5628	4.0882	4.9717	2.6404	1.1135	0.5233
	0.10	1.0031	1.8866	2.3066	3.5605	5.4883	3.5106	1.3557	0.5877
	0.01	1.0207	2.1693	3.1531	4.1496	3.2193	1.6558	0.7839	0.3915
1.50	0.05	1.0061	1.9518	2.5825	4.2292	5.1740	2.5722	1.0343	0.4691
	0.10	1.0030	1.8851	2.3069	3.6123	5.7735	3.4555	1.2833	0.5318

$$\hat{\theta}_4 = d_1 \frac{T_r}{r} + \left(\left(1 - d_1\right) \theta_0\right) I_{(t_1 \le T_r \le t_2)}. \tag{2.10}$$

The risk of the proposed shrinkage testimator $\hat{\theta}_4$ under ILL is given by

$$\begin{aligned} \text{Risk}\left(\hat{\theta}_4\right) = & \left\{ e^{c(\delta-1)} G\left(w_1, w_2, e^{f_3}\right) - e^{-c} G\left(w_1, w_2, f_0\right) - c\delta\left(1 + d_1\right) G\left(w_1, w_2, 1\right) \right. \\ & \left. + c + (1 + r)\left(\exp\left(-\frac{c}{r+1}\right) - 1\right) \right\}, \end{aligned} \tag{2.11}$$

Table 3
$RE\left(\hat{\theta}_3, Y_1\right)$

a	α	δ							
		0.04	0.06	0.80	1.00	1.20	1.40	1.60	1.80
$r = 06$									
	0.01	1.0450	2.2791	3.4407	5.5356	4.5836	2.3385	1.2709	0.7922
0.25	0.05	1.0283	2.2217	3.3252	4.6326	3.9400	2.3587	1.4332	0.9639
	0.10	1.0368	2.1993	3.0612	4.1342	3.7142	2.3966	1.5319	1.0725
	0.01	1.0354	2.2619	3.5240	5.6140	4.4927	2.2410	1.2011	0.7398
0.50	0.05	1.0456	2.2893	3.3808	4.7780	3.9802	2.3013	1.3658	0.9038
	0.10	1.0362	2.1987	3.0850	4.2446	3.7979	2.3770	1.4789	1.0152
	0.01	1.0473	2.3365	3.6997	5.8099	4.3287	2.0566	1.0696	0.6417
1.00	0.05	1.0461	2.3036	3.4809	5.0870	4.0716	2.1900	1.2371	0.7908
	0.10	1.0349	2.1945	3.1204	4.4592	3.9715	2.3348	1.3734	0.9045
	0.01	1.0563	2.4023	3.8845	6.0600	4.1865	1.8858	0.9486	0.5526
1.50	0.05	1.0460	2.3114	3.5643	5.4147	4.1749	2.0823	1.1165	0.6870
	0.10	1.0336	2.1874	3.1407	4.6591	4.1487	2.2869	1.2689	0.7994
$r = 08$									
	0.01	1.0125	2.0783	3.0981	4.9944	3.9823	1.9684	1.0761	0.6851
0.25	0.05	1.0201	2.1157	3.0206	4.2888	3.5364	2.0349	1.2376	0.8529
	0.10	1.0143	2.0372	2.7339	3.7674	3.4052	2.1277	1.3473	0.9588
	0.01	1.0156	2.1041	3.1642	5.0847	3.9259	1.8953	1.0216	0.6428
0.50	0.05	1.0200	2.1180	3.0502	4.4041	3.5754	1.9912	1.1844	0.8040
	0.10	1.0140	2.0345	2.7415	3.8426	3.4782	2.1157	1.3069	0.9137
	0.01	1.0208	2.1502	3.2983	5.2940	3.8237	1.7557	0.9179	0.5631
1.00	0.05	1.0197	2.1198	3.1002	4.6406	3.6596	1.9052	1.0816	0.7108
	0.10	1.0132	2.0284	2.7499	3.9833	3.6267	2.0880	1.2251	0.8250
	0.01	1.0246	2.1893	3.4325	5.5431	3.7349	1.6248	0.8215	0.4898
1.50	0.05	1.0192	2.1187	3.1378	4.8808	3.7502	1.8203	0.9840	0.6241
	0.10	1.0125	2.0214	2.7502	4.1083	3.7756	2.0546	1.1424	0.7388
$r = 10$									
	0.01	1.0073	2.0077	2.9048	4.6357	3.5317	1.7111	0.9477	0.6178
0.25	0.05	1.0087	2.0069	2.7842	4.0464	3.2900	1.8294	1.1143	0.7862
	0.10	1.0053	1.9363	2.4735	3.4694	3.2331	1.9772	1.2367	0.8892
	0.01	1.0085	2.0230	2.9560	4.7285	3.4946	1.6525	0.9023	0.5817
0.50	0.05	1.0085	2.0059	2.7973	4.1361	3.3284	1.7945	1.0700	0.7444
	0.10	1.0051	1.9338	2.4736	3.5190	3.3006	1.9721	1.2056	0.8532
	0.01	1.0104	2.0496	3.0564	4.9356	3.4271	1.5401	0.8157	0.5132
1.00	0.05	1.0082	2.0030	2.8170	4.3142	3.4087	1.7248	0.9839	0.6639
	0.10	1.0048	1.9286	2.4803	3.6083	3.4363	1.9580	1.1413	0.7807
	0.01	1.0117	2.0711	3.1526	5.1717	3.3681	1.4336	0.7345	0.4497
1.50	0.05	1.0079	1.9991	2.8287	4.4873	3.4922	1.6550	1.0009	0.5881
	0.10	1.0045	1.9234	2.4773	3.6835	3.5706	1.9381	1.0746	0.7084

where $f_3 = \frac{cd_1}{m_2 - m_1}\left(\frac{w}{r} - \delta\right)$.

The relative efficiency for $\hat{\theta}_i$; $i = 1, --, 4,$ with respect to the minimum risk improved estimator under the ILL is defined as

$$RE\left(\hat{\theta}_i, Y_1\right) = \frac{\text{Risk}\,(Y_1)}{\text{Risk}\left(\hat{\theta}_i\right)}; \quad i = 1, --, 4.$$

The expression for the relative efficiency $RE\left(\hat{\theta}_i, Y_1\right)$; $i = 1, --, 4,$ is the function of r, c, δ and α. For the selected values of $r = 06, 08, 10$; $c = 0.25, 0.50, 1.00, 1.50$; $\delta = 0.40(0.20)1.80$ and $\alpha = 0.01, 0.05, 0.10,$ the relative

Table 4
$RE\left(\hat{\theta}_4, Y_1\right)$

a	α	δ							
		0.04	0.06	0.80	1.00	1.20	1.40	1.60	1.80
$r = 06$									
	0.01	1.0873	2.3504	2.6361	2.5361	2.2974	2.0955	1.9396	1.8047
0.25	0.05	1.0405	2.1496	2.6084	2.8918	2.7628	2.4390	2.1357	1.8992
	0.10	1.0249	2.0440	2.4653	2.9248	3.0503	2.7793	2.3863	2.0443
	0.01	1.0910	2.3977	2.7370	2.6144	2.3129	2.0572	1.8621	1.6984
0.50	0.05	1.0417	2.1701	2.6868	3.0270	2.8637	2.4578	2.0870	1.8061
	0.10	1.0254	2.0554	2.5176	3.0530	3.2037	2.8587	2.3745	1.9697
	0.01	1.0965	2.4829	2.9460	2.7766	2.3283	1.9554	1.6831	1.4700
1.00	0.05	1.0431	2.2029	2.8340	3.3098	3.0678	2.4712	1.9608	1.6004
	0.10	1.0260	2.0725	2.6094	3.3091	3.5245	3.0006	2.3156	1.7933
	0.01	1.0998	2.5552	3.1630	2.9487	2.3665	1.8695	1.4884	1.2403
1.50	0.05	1.0437	2.2264	2.9663	3.6078	3.2750	2.4750	1.8080	1.3868
	0.10	1.0262	2.0834	2.6842	3.5607	3.8632	3.1169	2.2168	1.5968
$r = 08$									
	0.01	1.0389	2.1679	2.5071	2.4800	2.2489	2.0508	1.9121	1.8017
0.25	0.05	1.0148	1.9954	2.3793	2.7129	2.6651	2.3767	2.0962	1.8862
	0.10	1.0081	1.9242	2.2400	2.6704	2.8694	2.6725	2.3226	2.0143
	0.01	1.0404	2.1961	2.5886	2.5582	2.2723	2.0254	1.8520	1.7159
0.50	0.05	1.0151	2.0059	2.4303	2.8223	2.7628	2.4055	2.0638	1.8129
	0.10	1.0083	1.9298	2.2718	2.7632	3.0014	2.7541	2.3207	1.9587
	0.01	1.0426	2.2452	2.7531	2.7208	2.3080	1.9530	1.7385	1.5248
1.00	0.05	1.0156	2.0225	2.5234	3.0463	2.9630	2.4448	1.9728	1.6443
	0.10	1.0085	1.9380	2.3262	2.9440	3.2763	2.9079	2.3003	1.8205
	0.01	1.0438	2.2852	2.9176	2.8935	2.3306	1.8576	1.5453	1.3235
1.50	0.05	1.0158	2.0341	2.6042	3.2758	3.1699	2.4610	1.8549	1.4606
	0.10	1.0085	1.9433	2.3696	3.1159	3.5649	3.0472	2.2410	1.6584
$r = 10$									
	0.01	1.0166	2.0403	2.3942	2.4472	2.2294	2.0279	1.8964	1.8005
0.25	0.05	1.0052	1.9077	2.2179	2.5751	2.6026	2.3443	2.0745	1.8790
	0.10	1.0026	1.8618	2.0957	2.4886	2.7419	2.6075	2.2975	1.9965
	0.01	1.0172	2.0570	2.4588	2.5245	2.2592	2.0113	1.8478	1.7287
0.50	0.05	1.0053	1.9133	2.2518	2.6640	2.6971	2.3799	2.0529	1.8189
	0.10	1.0026	1.8646	2.1157	2.5575	2.8572	2.6899	2.2970	1.9533
	0.01	1.0180	2.0854	2.5856	2.6855	2.3009	1.9503	1.7288	1.5646
1.00	0.05	1.0055	1.9219	2.3123	2.8427	2.8919	2.4377	1.9863	1.6769
	0.10	1.0027	1.8687	2.1493	2.6888	3.0955	2.8502	2.2956	1.8417
	0.01	1.0185	2.1079	2.7083	2.8559	2.3029	1.8465	1.5893	1.3861
1.50	0.05	1.0056	1.9278	2.3632	3.0209	3.0950	2.4474	1.8942	1.5169
	0.10	1.0027	1.8713	2.1757	2.8104	3.3432	3.0030	2.2644	1.7054

efficiencies have been calculated and presented in Tables 1–4. Only positive values of c are considered because overestimation in mean life is more serious than the underestimation.

From these tables it is observed that the shrinkage testimators $\hat{\theta}_1$ and $\hat{\theta}_4$ perform better than the improved estimator Y_1 for all considered values of r, c, δ and α. The testimators $\hat{\theta}_2$ and $\hat{\theta}_3$ perform better than Y_1 in $0.40 \leq \delta \leq 1.40$ for all selected values of level of significance α and in $0.40 \leq \delta \leq 1.60$ when $\alpha > 0.01$. The testimators $\hat{\theta}_1$ and $\hat{\theta}_3$ attain maximum efficiency at the point $\delta = 1.00$ and others near to the point $\delta = 1.00$.

For fixed c and level of significance α, as the uncensored sample size r increases, the relative efficiency decreases in the region $0.40 \leq \delta \leq 1.00$ for the testimators $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_4$, and for testimator $\hat{\theta}_3$ it decreases for all considered values of δ.

For fixed r and α, when c increases the relative efficiency decreases when $\delta \geq 1.40$ for all testimators. It has been seen that as the level of significance α increases the relative efficiency decreases first for small $\delta \leq 0.80$ (except $\hat{\theta_3}$ and $\hat{\theta_4}$) and then increases for large $\delta \geq 1.40$.

3. Recommendations

The recommendations have been presented, based on the relative efficiency for all the shrinkage testimator. The testimator $\hat{\theta_1}$ performs well when $\delta = 1$ for $\alpha > 0.01$. The shrinkage testimator $\hat{\theta_2}$ is preferable when $1.20 \leq \delta \leq 1.40$ and for small $\alpha = 0.01$ when $0.40 \leq \delta \leq 0.60$. The testimator $\hat{\theta_3}$ is referable when $\delta = 0.80$ and $\delta = 1.00$ for small $\alpha = 0.01$. Otherwise, it is safe to use $\hat{\theta_4}$ for $\delta \geq 1.40$.

The shrinkage testimators $\hat{\theta_1}$ and $\hat{\theta_4}$ always perform better than other shrinkage testimators if the gain in efficiency does not matter. The testimator $\hat{\theta_1}$ is preferable when $0.80 \leq \delta \leq 1.20$ and $\hat{\theta_4}$ otherwise.

4. Displaced exponential model and shrinkage testimators

The probability density function of displaced exponential distribution for any random variable x is

$$f(x;\theta,a) = \begin{cases} \dfrac{1}{\theta}\mathrm{e}^{-\left(\frac{x-a}{\theta}\right)}; & x \geq a, \theta \geq 0 \\ 0; & \text{otherwise.} \end{cases}$$

If type-II censored data is available then the total test time is

$$S_r = \sum_{j=1}^{r} \left(x_{(j)} - x_{(1)}\right) + (n-r)\left(x_{(r)} - x_{(1)}\right); \quad n > r \tag{4.1}$$

and the unbiased estimate for θ is $\frac{S_r}{r}$. A test statistic $\frac{2S_r}{\theta} \backsim \chi^2_{2(r-1)}$ is available for testing the hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

In case one is interested in studying the properties of the proposed shrinkage testimators in displaced exponential type-II censored data under ILL criterion, he may take only a specific change $(r-1)$ in place of r.

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