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Empirical Bayes estimators of reliability performances using LINEX loss under progressively Type-II censored samples

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Abstract

Based on progressively Type-II censored samples, the empirical estimators of reliability performances for Burr XII distribution are researched under LINEX error loss. Firstly, we obtain the Bayes estimators of the reliability performances. Secondly, different from the predecessor, the empirical Bayes estimators of the reliability performances are derived where hyper-parameter is estimated using maximum likelihood method. In the end, in order to investigate the accuracy of estimations, an illustrative example is examined numerically by means of Monte-Carlo simulation.

Keywords: Empirical Bayes estimate; Burr XII model; Progressively censored samples; Asymmetric loss function

1. Introduction

The two-parameter Burr Type-XII distribution has already gained special attention in the literature since Burr first introduced it. The probability density function and cumulative distribution function of the Burr (c, k) distribution are given, respectively, by

$$f(x; c, k) = ckx^{c-1}(1 + x^c)^{-(k+1)}, \quad x > 0,\ c > 0,\ k > 0 \tag{1}$$

$$F(x; c, k) = 1 - (1 + x^c)^{-k}, \quad x > 0 \tag{2}$$

where c and k are all shape parameters.

Inferences for Burr XII model were discussed by many authors. Reference [14] presented statistical and probabilistic properties of the Burr XII distribution and described its relationship to other distributions used in reliability analyses. Moreover, the author pointed out that the Burr XII could cover the curve shape characteristics for the normal, Weibull, logistic, lognormal and Extreme Value Type-I distributions. Based on censored data as well as complete data, using the maximum likelihood method, reference [12] gave the method for obtaining point and interval estimates of the parameters of the Burr XII distribution. Under squared error loss function, Bayes approximate estimates and maximum likelihood estimates for the two parameters and the reliability function of the Burr XII distribution have been obtained

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based on progressive Type-II censored samples in reference [3], while reference [7] derived Bayesian estimates of the parameter k and the reliability function under three different loss functions. Based on the same progressive samples as above, reference [11] obtained the Bayes estimators using both the symmetric loss function and asymmetric loss function.

But up to now, the empirical Bayes estimates related to the Burr XII were not addressed under progressively censored sample which has been described particularly in references [1,4]. So in this paper, under asymmetric loss function, the empirical Bayes estimator of shape parameter for the Burr XII distribution is derived based on progressively Type-II censored data, at the same time; we give the empirical Bayes estimators of the reliability function and the failure rate for reliability assessment in the engineering.

We assume that c is known; k has a gamma conjugate prior density $\Gamma(1, \beta)$

$$\pi(k; \beta) = \beta \exp(-\beta k), \quad k > 0,\ \beta > 0$$

That is to say, we regard random variable k with prior density as exponential distribution $\exp(\beta)$, which is usually used in Bayesian theory (see [9,13]).

2. Bayes estimators of the reliability performances

We design an experiment in which n units are placed on the test at the beginning time, and this test can be terminated at any failure time. Suppose that all units are independent and have identical Burr XII distribution (1). The progressively Type-II censored test steps are as follows.

When a working unit fails, we refer to the first failure time as $X_{1,n}$ and remove r_1 units from the remaining $n-1$ units. That is, at the time of the first failure $X_{1,n}$, r_1 units are randomly removed from the remaining $n-1$ surviving units. Similarly, at the second failure time $X_{2,n}$, r_2 units from the remaining $n-2-r_1$ units are randomly removed. The test continues until the mth failure. At this time, all remaining $r_m = n-m-r_1-r_2 \cdots - r_{m-1}$ units are removed. In this censoring scheme, r_i and m are pre-fixed.

In this test, we can see that when $r_1 = r_2 = \cdots = r_m = 0$, it reduces to the case of no censoring (complete sample case) and when $r_1 = r_2 = \cdots = r_{m-1} = 0$, it reduces to a Type-II censored sample.

Suppose that $X = (X_{1,m,n}, X_{2,m,n}, \ldots, X_{m,m,n})$ is a progressively Type-II censored sample from a life test on items whose lifetimes have Burr (c, k) distribution (1). The likelihood function based on above samples (see [4]) is given by

$$L(k|x) = A\prod_{i=1}^{m} f(x_{i,m,n}; c, k)[1 - F(x_{i,m,n}; c, k)]^{r_i} \tag{3}$$

where $A = n(n-1-r_1)(n-2-r_1-r_2)\cdots(n - \sum_{i=1}^{m-1}(r_i+1))$.

From (1) and (2), function L is

$$L(k|x) = A\prod_{i=1}^{m} kcx_i^{c-1}(1 + x_i^c)^{-(k(r_i+1)+1)} \tag{4}$$

where $x_i \equiv x_{i,m,n}$.

From Bayesian theorem, the posterior distribution of k can be written as

$$\pi^*(k|T) = \frac{\pi(k;\beta)L(k|x)}{\int_0^{+\infty} \pi(k;\beta)L(k|x)\,\mathrm{d}k} = \frac{\beta \exp(-\beta k)A\prod_{i=1}^{m} kcx_i^{c-1}(1+x_i^c)^{-(k(r_i+1)+1)}}{\int_0^{+\infty} \beta \exp(-\beta k)A\prod_{i=1}^{m} kcx_i^{c-1}(1+x_i^c)^{-(k(r_i+1)+1)}\,\mathrm{d}k}$$
$$= \frac{(\beta+T)^{m+1}k^m \exp(-k(\beta+T))}{\Gamma(m+1)} \tag{5}$$

where $T = \sum_{i=1}^{m}(r_i+1)\ln(1+x_i^c)$.

Reference [8] pointed out that, in some situations, the use of asymmetric loss functions may be appropriate, so a very useful asymmetric loss function known as LINEX loss function has been proposed and adopted by some researchers (see[6]) in recent years. It can be expressed as

$$L(\tilde{u} - u) = b(\mathrm{e}^{a(\tilde{u}-u)} - a(\tilde{u} - u) - 1), \quad a \neq 0,\ b > 0 \tag{6}$$

where $\tilde{u}$ is an estimation of u.

From (6), we can see that, when $a>0$, overestimation is more serious than underestimation; however, when $a<0$, the conclusion is opposite. As 'a' nears to zero, the LINEX loss function is approximately the squared error loss, and therefore almost symmetric.

So the posterior-expectation of (6) is

$$E_{\text{post}}(L(\tilde{u}-u)) = b(\mathrm{e}^{a\tilde{u}} E_{\text{post}}(\mathrm{e}^{-au}) + aE_{\text{post}}(u) - a\tilde{u} - 1), \quad a \neq 0,\ b > 0 \tag{7}$$

The value of u that minimizes (7), denoted by $\tilde{u}$, is obtained as follows:

$$\tilde{u} = -\frac{1}{a}\ \ln E_{\text{post}}\, \mathrm{e}^{-au} \tag{8}$$

So the Bayes estimation of k is

$$\tilde{k} = -\frac{1}{a}\ \ln \int_0^{+\infty} \mathrm{e}^{-ak}\, \pi^*(k|T)\,\mathrm{d}k = \frac{m+1}{a}\ \ln\left(1+\frac{a}{\beta+T}\right) \tag{9}$$

As every unit has Burr (c, k) distribution, the reliability function $R(t)$ and the failure rate $\lambda(t)$ at time t are given by

$$R(t) = (1+t^c)^{-k} \tag{10}$$

$$\lambda(t) = -\frac{R'(t)}{R(t)} = \frac{ckt^{c-1}}{1+t^c} \tag{11}$$

Thus, the Bayes estimators of $R(t)$ and $\lambda(t)$ at time t under LINEX loss function are

$$\begin{aligned}\tilde{R}(t) &= -\frac{1}{a}\ \ln \int_0^{+\infty} \mathrm{e}^{-a(1+t^c)^{-k}}\, \pi^*(k|T)\,\mathrm{d}k = -\frac{1}{a}\ \ln \int_0^{+\infty} \sum_{s=0}^{\infty} \frac{[-a(1+t^c)^{-k}]^s}{s!} \pi^*(k|T)\,\mathrm{d}k \\ &= -\frac{1}{a}\ \ln \sum_{s=0}^{\infty} \frac{(-a)^s}{s!}\left[1+\frac{s\ \ln(1+t^c)}{\beta+T}\right]^{-(m+1)}\end{aligned} \tag{12}$$

$$\tilde{\lambda}(t) = -\frac{1}{a}\ \ln \int_0^{+\infty} \mathrm{e}^{-a(ckt^{c-1}/(1+t^c))}\, \pi^*(k|T)\,\mathrm{d}k = \frac{m+1}{a}\ \ln\left(1+\frac{act^{c-1}}{(1+t^c)(\beta+T)}\right) \tag{13}$$

3. Empirical Bayes estimators of the reliability performances

In theory, the accuracy of maximum likelihood estimation is higher than that of previous estimation (moment estimation). In view of this fact, reference [13] used the maximum likelihood method to estimate hyper-parameter of prior distribution for analyzing the Bayesian reliability quantitative indexes of cold standby system.

In (9), the hyper-parameter β is an unknown constant, so $\tilde{k}$ cannot be used directly. Therefore, we make use of the maximum likelihood method to estimate β.

As all units have identical Burr XII distribution Burr (c, k), the margin density function is

$$f(x) = \int_0^{+\infty} f(x;c,k)\pi(k;\beta)\,\mathrm{d}k = \int_0^{+\infty} ckx^{c-1}(1+x^c)^{-(k+1)}\beta\ \exp(-\beta k)\,\mathrm{d}k = \frac{\beta c x^{c-1}}{(1+x^c)(\beta+\ln(1+x^c))^2}$$

$$1-F(x) = \int_x^{+\infty} f(x)\,\mathrm{d}x = \int_x^{+\infty} \frac{\beta c x^{c-1}}{(1+x^c)(\beta+\ln(1+x^c))^2}\,\mathrm{d}x = \frac{\beta}{\beta+\ln(1+x^c)}$$

Hence, (3) can be expressed as

$$L(\beta|X) = A\prod_{i=1}^{m} f(x_{i,m,n})[1-F(x_{i,m,n})]^{r_i} \tag{14}$$

Substituting $f(x)$ and $F(x)$ into (14), function L is

$$L(k|x) = A\prod_{i=1}^{m}\frac{\beta c x_i^{c-1}}{(1+x_i^c)(\beta+\ln(1+x_i^c))^2}\left[\frac{\beta}{\beta+\ln(1+x_i^c)}\right]^{r_i}$$

$$\ln L = \ln A + m(\ln c + \ln\beta) + \sum_{i=1}^{m}\left[\ln\frac{x_i^{c-1}}{1+x_i^c} - \ln((\beta+\ln(1+x_i^c))^2 + \ln\beta^{r_i} - \ln(\beta+\ln(1+x_i^c))^{r_i}\right]$$

$$\frac{\mathrm{d}\ln L}{\mathrm{d}\beta} = \frac{m}{\beta} - 2\sum_{i=1}^{m}\frac{1}{\beta+\ln(1+{x_i}^c)} + \sum_{i=1}^{m} r_i\left(\frac{1}{\beta} - \frac{1}{\beta+\ln(1+{x_i}^c)}\right)$$

Consider function

$$g_1(\beta) = \frac{m}{\beta} + \sum_{i=1}^{m} r_i\left(\frac{1}{\beta} - \frac{1}{\beta+\ln(1+x_i^c)}\right), \qquad g_2(\beta) = 2\sum_{i=1}^{m}\frac{1}{\beta+\ln(1+x_i^c)}$$

As the MLE of β is needed, we just draw a conclusion that equation $g_1(\beta)=g_2(\beta)$ has only one root. The reasons are

$$g_1(\beta) > 0, \quad g_1(\beta) \to 0(\beta\to\infty), \quad g_1(\beta)\to\infty(\beta\to 0)$$

$$g_1'(\beta) = -\left\{m\beta^{-2} + \sum_{i=1}^{m} r^i\ \ln(1+x_i^c)(2\beta+\ln(1+x_i^c))[\beta^2(\beta+\ln(1+x_i^c))^2]^{-1}\right\} < 0$$

$$g_1''(\beta) = 2m\beta^{-3} + 2\sum_{i=1}^{m} r^i\ \ln(1+x_i^c)(3\beta^2 + 3\beta\ \ln(1+x_i^c) + \ln^2(1+x_i^c))[\beta^3(\beta+\ln(1+x_i^c))^3]^{-1} > 0$$

So $g_1(\beta)$ is strict monotone increasing concave function. Similarly

$$g_2(\beta) > 0, \quad g_2(\beta) \to 0(\beta\to\infty), \quad g_2(\beta)\to\infty(\beta\to 0)$$

$$g_2'(\beta) = -2\sum_{i=1}^{m}(\beta+\ln(1+x_i^c))^{-2} < 0$$

$$g_2''(\beta) = 4\sum_{i=1}^{m}(\beta+\ln(1+x_i^c))^{-3} > 0$$

Therefore, $g_2(\beta)$ is strict monotone increasing concave function. Moreover

$$\lim_{\beta\to\infty}\frac{g_1(\beta)}{g_2(\beta)} = \lim_{\beta\to\infty}\left[\frac{m}{\beta} + \sum_{i=1}^{m} r_i\left(\frac{1}{\beta} - \frac{1}{\beta+\ln(1+x_i^c)}\right)\right]\left[2\sum_{i=1}^{m}\frac{1}{\beta+\ln(1+x_i^c)}\right]^{-1} = \frac{1}{2}$$

From above, the equation $\mathrm{d}\ln L/\mathrm{d}\beta=0$ has only one root (see [13]), and from the equation $\mathrm{d}\ln L/\mathrm{d}\beta=0$, we can get

$$\beta = m\left[2\sum_{i=1}^{m}\frac{1}{\beta+\ln(1+x_i^c)} - \sum_{i=1}^{m} r_i\left(\frac{1}{\beta} - \frac{1}{\beta+\ln(1+x_i^c)}\right)\right]^{-1}$$

Using iterative computing method to obtain the solution, the iteration formula is

$$\beta^{(l+1)} = m\left[2\sum_{i=1}^{m}\frac{1}{\beta^{(l)}+\ln(1+{x_i}^c)} - \sum_{i=1}^{m} r_i\left(\frac{1}{\beta^{(l)}} - \frac{1}{\beta^{(l)}+\ln(1+x_i^c)}\right)\right]^{-1}, \quad l = 1, 2, 3, \ldots \tag{15}$$

where $\beta^{(l)}$ is lth iterative value ($l = 1, 2, 3, \ldots$), $\beta^{(1)}$ is an initial value.

If the iteration solution is denoted by $\hat{\beta}$, then the empirical estimation of $\tilde{k}$ is

$$\hat{k} = \frac{m+1}{a} \ln\left(1 + \frac{a}{\hat{\beta}+T}\right) \tag{16}$$

where β is replaced by $\hat{\beta}$.

Substituting $\hat{\beta}$ into (12), the empirical estimation of $\tilde{R}(t)$ is obtained

$$\hat{R}(t) = -\frac{1}{a} \ln \sum_{s=0}^{\infty} \frac{(-a)^s}{s!} \left[1 + \frac{s \ln(1+t^c)}{\hat{\beta}+T}\right]^{-(m+1)} \tag{17}$$

Similarly, the empirical Bayes estimation of $\tilde{\lambda}(t)$ is given as follows:

$$\hat{\lambda}(t) = \frac{m+1}{a} \ln\left(1 + \frac{act^{c-1}}{(1+t^c)(\hat{\beta}+T)}\right) \tag{18}$$

4. An example

We firstly generate progressive Type-II censored samples from Burr (c, k) distribution. Applying the algorithms of Balakrishnan and Sandhu [5] and Aggarwala and Balakrishnan [2], the steps are:

(A) Generate m independent $U(0, 1)$ random variables $W_1, W_2, \ldots, W_m$.
(B) For given values of the progressive censoring scheme $r_1, r_2, \ldots r_m$, set

$$V_i = W_i^{1/(i+r_m+r_{m-1}+\cdots+r_{m-i+1})}, \quad i = 1, 2, \ldots, m.$$

(C) Set $U_i = 1 - (V_m V_{m-1} \ldots V_{m-i+1})$, $i = 1, 2, \ldots, m$; then $U_1, U_2, \ldots, U_m$ are progressive Type-II censored samples of size m from $U(0, 1)$.
(D) Thus, for given values of parameters c and k, $x_i = [(1-U_i)^{-1/k} - 1]^{1/c}$, $i = 1, 2, \ldots m$, is the required progressive Type-II censored sample of size m from Burr (c, k) distribution.

The empirical Bayes estimators of β, k, $R(t)$ and $\lambda(t)$ are derived by means of Monte-Carlo simulation. The steps are as follows in detail:

(1) For given value of β, a group of values of k is generated according to $\pi(k; \beta) = \beta \exp(-\beta k)$. Taking one of them as k^*, and substituting k^* into (10) and (11), $R^*(t)$ and $\lambda^*(t)$ can be obtained.
(2) For given c, k^*, m, n and $(r_1, r_2, \ldots, r_m)$, $(x_1, x_2, \ldots, x_m)$ can be derived via above-mentioned method.
(3) Combining (1) and (2) with given values, we can get $\hat{\beta}$ according to (15).
(4) For given t, $\hat{k}$, $\hat{R}(t)$ and $\hat{\lambda}(t)$ are obtained, respectively, by (16), (17) and (18).

For 1000 repetitions, the estimated risks (ER) of the different estimators are computed as the average of their squared deviations. The expression is

$$1000^{-1} \sum_{i=1}^{1000} (\hat{q} - q^*)^2$$

where $\hat{q}$ denotes $\hat{\beta}$, $\hat{k}$, $\hat{R}(t)$ and $\hat{\lambda}(t)$, while q^* denotes $\beta^* = 2$, k^*, $R^*(t)$ and $\lambda^*(t)$.

Tables 2 and 3 display the estimated risks of the estimates of β, k, $R(t)$ and $\lambda(t)$ under small-scale simulation.Three different cases of the sample size and censoring scheme are shown in Table 1. Moreover, Table 4 gives us the estimated risks of the estimates in large samples.

Table 1
Censoring scheme $(r_1, r_2, \ldots, r_m)$

Sample size	$(r_1, r_2, \ldots, r_m)$
$n=20$, $m=10$	0 2 1 0 1 1 2 0 0 3
$n=30$, $m=20$	0 1 0 0 0 0 2 0 0 0 2 0 0 3 0 0 1 0 0 1
$n=40$, $m=30$	1 0 2 0 0 1 0 2 0 0 0 1 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 1

Table 2
Estimated risks (ER) of the estimates of β, k, $R(t)$ and $\lambda(t)$ ($a=0.5$, $t=2$, $c=3$, $\beta=2$)

Sample size (n, m)	ER, $\hat{\beta}$	ER, $\hat{k}$	ER, $\hat{R}(2)$	ER, $\hat{\lambda}(2)$
$n=20$, $m=10$	0.4682	0.0228	0.0511	0.1386
$n=30$, $m=20$	0.2376	0.0153	0.0276	0.1017
$n=40$, $m=30$	0.2058	0.0111	0.0048	0.0619

Table 3
Estimated risks (ER) of the estimates of β, k, $R(t)$ and $\lambda(t)$ ($a=-1.5$, $t=3$, $c=3$, $\beta=2$)

Sample size (n, m)	ER, $\hat{\beta}$	ER, $\hat{k}$	ER, $\hat{R}(3)$	ER, $\hat{\lambda}(3)$
$n=20$, $m=10$	0.7010	0.0459	0.0703	0.3386
$n=30$, $m=20$	0.6257	0.0126	0.0696	0.1017
$n=40$, $m=30$	0.4351	0.0121	0.0027	0.0519

Table 4
Estimated risks (ER) of the estimates of β, k, $R(t)$ and $\lambda(t)$ ($a=1$, $t=5$, $c=3$, $\beta=2$)

Sample size (n, m)	ER, $\hat{\beta}$	ER, $\hat{k}$	ER, $\hat{R}(5)$	ER, $\hat{\lambda}(5)$
$n=100$, $m=90$	0.2032	0.0108	0.0025	0.0496
$n=80$, $m=80$	0.1873	0.0101	0.0019	0.0381
$n=100$, $m=100$	0.1634	0.0098	0.0017	0.0329
$n=200$, $m=200$	0.1205	0.0092	0.0014	0.0162

5. Conclusions

(1) The Burr model can be widely used in reliability applications because it has many different forms of its reliability and hazard functions. It gives the reliability practitioner another model for representing failure data. Soliman [10] told us that it has been applied in areas of quality control, duration and failure time modeling.

(2) Censored life testing plays an important role in reliability studies. The progressively Type-II censored scheme is a general one. We can see that when $r_1=r_2=\cdots=r_m=0$, it reduces to the complete sample case and when $r_1=r_2=\cdots=r_{m-1}=0$, it reduces to Type-II censored sample.

(3) For $a=0.5$ and $t=2$, the estimated risks of $\hat{\beta}$, $\hat{k}$, $\hat{R}(2)$ and $\hat{\lambda}(2)$ are displayed in Table 2, while, for different given values, all the estimated risks are obtained in Table 3. From the data in table, when m/n increases, the estimated risk of the estimates decreases.

(4) Tables 2 and 3 use the same sample size n, m and censoring scheme $(r_1, r_2, \ldots, r_m)$. The data are given in Table 1. In fact, different values of the sample size and the censoring scheme do not change the previous conclusions.

(5) Table 4 displays the estimated risks of the estimates in large samples, and, as anticipated, the estimated risks of the estimates get smaller with increasing samples.

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