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On the measurement of wall shear stress

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Abstract. The differential pressure reading from a static hole pair is utilized for determination of the local wall shear stress. Both the hole diameter and forward-facing angle are varied to test the sensitivity of the device. The static hole pair in tested in a two-dimensional zero pressure gradient turbulent boundary layer on a smooth surface. The calibrating values for the local wall shear is determined from the universal scaling laws for the mean velocity profile in the inner part of the turbulent boundary layer. The static hole pair is found to be sensitive to imperfections in the manufacturing process, and needs an individual calibration in order to make accurate measurements of the local skin friction possible.

1 Introduction

The the wall shear stress is a quantity which in many situations appears as a crucial parameter in the transport of mass and energy in ducts or in the neighbourhood of fixed walls. Most of the methods available for direct or indirect measurements of the wall shear stress in a turbulent boundary layer are reviewed by Winter (1977). Discrepancies between results in different experimental facilities can often be traced back to uncertainties in the measurement of the wall shear stress. Examples of discrepancies of 10% between data obtained in one and the same facility have been reported (Kline et al. 1967) when using different methods for the determination of the wall shear stress.

The object of the present work is to apply a method where the wall shear stress may be determined from measurements of the wall static pressure. Such a method has obvious advantages because the measuring principle is simple and because no measuring device that can disturb the flow field is needed. Shaw (1960) showed that as the diameter of a static pressure hole is increased the accuracy with which it measures the static pressure is reduced. The error is claimed to be due to the local skin friction in the boundary layer. Rajaratnam (1966) and Duffy and Norbury (1968) utilized the effect that two static pressure holes of different sizes give different readings for the static pressure for indirect measurements of the local wall shear stress. A variant on such a static hole pair was further explored by Green and Coleman (1973) which used the pressure difference between a pair of slots, one inclined 45° forwards and the other 45° backwards. The device produced a larger pressure difference than two static holes drilled perpendicular to the wall surface. Their aim was to produce a device the readings of which depending entirely on the wall shear stress, so that calibration could be universal for any flow. Unfortunately, the effects of local pressure gradients and inaccuracies in their device prevented an absolute calibration. The latest contribution in this field known to the authors is a presentation, given at the conference on Two-Phase-Annular and Dispersed Flows, Pisa 1984 (Kvernvold et al. 1984), where the difference in static pressure was used for determination of the wall shear stress in two-phase flows.

The present approach is to combine the two effects of a static hole pair with different hole diameters and different inclinations, and to assess the method as a device for measuring the local wall shear stress. The method is an indirect method and calibration against a method generally accepted for reliability in skin friction measurements is thus needed. In the present work the universal scaling laws for the mean velocity profile in the inner part of the turbulent boundary layer is utilized for determining the wall shear stress. This method is generally accepted for reliability (e.g. Coles 1962) as long as the flow conditions are well controlled and documented. Bradshaw (1964) demonstrated that substantial transverse variations of the wall shear stress can result from comparatively weak cross-flows. The measurements are performed in the 500 mm × 1,000 mm test section of a closed-return wind tunnel with well documented two-dimensional mean flow conditions and a zero longitudinal pressure gradient turbulent boundary layer developing on a flat plate (Saetran 1985). This was done to reduce the number of variables as far as possible.

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2 Arrangement for measuring static pressure difference

The idea behind the device is to utilize the pressure difference between two static pressure holes in a plane surface beneath a turbulent boundary layer for determination of the wall shear stress. The static pressure reading is dependent on the size of the static pressure hole (e.g. Shaw 1960) and on the inclination of the hole with respect to this plane surface.

A 200 mm × 670 mm acryl plate of 16 mm thickness is installed in the 1,000 mm × 500 mm wind tunnel test section and mounted flush with the test section floor. The test plate is equipped with 16 static pressure hole pairs. All 32 pressure holes are located side by side such that the center of all holes are 1,745 mm downstream the boundary layer trip. The tripping strip ensures a stable transition to the turbulent boundary layer. The flow conditions at one hole are thus not influenced by disturbances from the other holes.

A pair of static pressure holes consists of a hole A and a hole B with a 10 mm separation distance between the centers. For all 16 hole pairs the axis of hole A is perpendicular to the plane surface of the test plate. The diameter of all 16 holes A is 1.0 mm. The hole depth/diameter ratio is 10, thus giving a ratio which is large enough to avoid the shallow hole error pressure reading effect reported by e.g. Livesey et al. (1962).

For 9 of the static hole pairs the angle between the test plate surface and the axis of the hole B is fixed, $\alpha = 20°$. The hole size effect on the pressure difference between holes A and B is studied by varying the diameter range of holes B from 1.0 to 10.0 mm. For the 7 other static hole pairs the diameter of holes B is fixed, $d = 2.0$ mm. For these pairs the effect on the pressure difference is studied by varying the angle between the test plate surface and the axis of holes B from 12.5° to 50°. The angle of all holes B is such that all holes are forward-facing.

The manufacturing process for producing a static pressure hole is importent due to the large influence of burrs and imperfections of the hole edges on the pressure reading (Shaw 1960; Franklin & Wallace 1970). The static pressure holes used in the present experiments are manufactured by using the following procedure: A hole is drilled in an approx. 150 mm long, 10 mm diameter acryl plug. The hole is drilled from one end using a drill giving the resulting diameter of the static pressure hole. The depth of the hole is about 15 diameters. Then, a larger drill is used to make a piercing hole by drilling from the other side of the acryl plug. A 10 mm hole is then drilled in the test plate with the required angle to the plate surface. The plug is inserted using special glue into the hole so that the end with the small hole diameter is protruding above the surface of the test plate. In this process, care is taken to ensure that the required angle between the hole axis and the plate surface is obtained. The part of the plug body protruding above the test plate surface was then machined by grinding until a plane and smooth surface of the test plate is obtained. The final operation was to slightly hone the inside of the pressure hole to ensure that no burr is left in the hole. A sketch of a pressure hole is shown in Fig. 1.

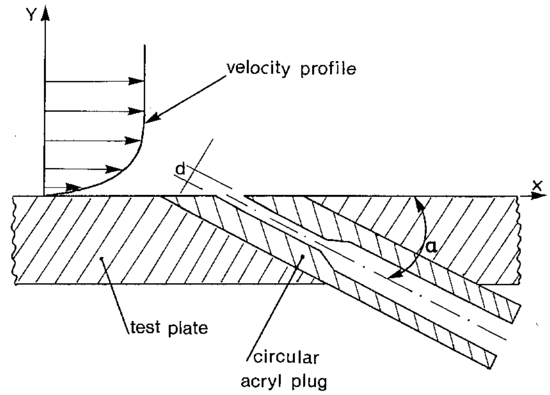

Fig. 1. Sketch of a static pressure hole

The pressure difference between hole A and hole B in a static pressure hole pair is measured by using a Wilh. Lambrecht KG Göttingen type 655 micromanometer giving an accuracy in pressure readings of 0.31 N/m^2 per division. The connections to the manometer were made by using 10 mm diameter rubber tubing.

3 Calibration values for the local wall shear stress

For the present calibration experiments the hot-wire anemometry is chosen for the velocity measurements. The mean velocity profile is measured by using a standard 5 µm diameter DISA 55P05 hot-wire operated by a 55M01 Constant Temperature Anemometer at an overheat ratio of 0.8. The signals are linearized in a 55M25 Linearizer and sampled by a Hewlett Packard computer system.

In the wall region of a turbulent boundary layer the mean velocity profile is well expressed through the Spalding (1961) formulation

$$y^+ = u^+ + A\,[\exp(\varkappa u^+) - 1 - (\varkappa u^+) - (\varkappa u^+)^2/2 - (\varkappa u^+)^3/6 - (\varkappa u^+)^4/24] \qquad (1)$$

where the constants A and $\varkappa$ are to be given the values

$$A = 0.015\,; \quad \varkappa = 0.53227\,. \qquad (2)$$

This particular choice of the constants is suggested by Persen (1978) in order to match experimental results in the wall region of a turbulent boundary layer, and is different from that of Spalding who found a best fit throughout the whole boundary layer with $\varkappa = 0.4$ and $A = 0.1108$.

Since the mean velocity $\bar{u}$ and the distance from the wall y are measured quantities and since both the non-dimensional quantities u^+ and y^+ are defined through the

Table 1. Values for the free-stream velocity and shear velocity values determined from the mean velocity profile

Case	Symbol	U_0 [m/s]	v_* [m/s]	v_* accuracy [%]
A	□	25.90	0.9431	$\pm$ 0.9
B	●	7.55	0.2893	$\pm$ 0.9
C	○	18.25	0.6535	$\pm$ 0.6
D	◇	22.20	0.7811	$\pm$ 1.0
E	■	13.30	0.4622	$\pm$ 1.3

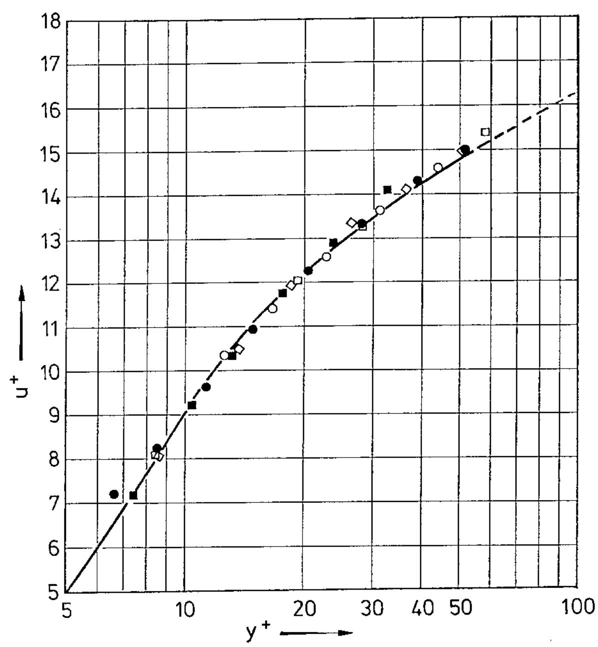

Fig. 2. u^+ versus y^+ in the wall region

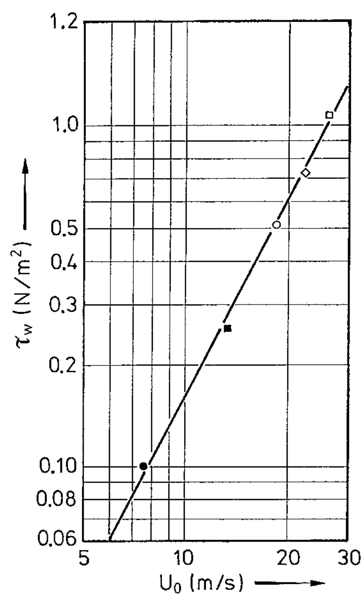

Fig. 3. Wall shear stress τ_ω as a function of the free-stream velocity U_0

shear velocity v_*

$$u^+ = \frac{\bar{u}}{v_*}; \quad y^+ = \frac{y\,v_*}{\nu}; \quad v_* = \left(\frac{\tau_\omega}{\varrho}\right)^{1/2} \tag{3}$$

a simple iteration procedure will determine the value of v_* (and thereby the local wall shear stress τ_ω) which for the measured values will satisfy Eq. (1).

Table 1 summarizes the data for v_* and the accuracy in finding v_* from Eq. (1) for 5 different cases of the free-stream velocity U_0 . It is seen that the mean values of the iterated v_*-values have a standard deviation of $\sim 1.0\%$ or less for each case. Figure 2 shows how the data (non-dimensionalized with the value of v_*) gather around the theoretical curve defined through Eqs. (1) and (2) with a satisfactory accuracy. Figure 3 shows the values for the wall shear stress τ_ω (determined from the v_*-values) as a function of the free-stream velocity U_0. The solid line is given by

$$\tau_\omega = 0.0020\, U_0^{1.911}\,. \tag{4}$$

This determines the wall shear stress against which the static pressure device is going to be tested.

4 Results and discussion

The experiments are conducted in the following way: The wind tunnel freestream speed is set to the required level, the mean velocity profile is measured by using the hot wire and the pressure difference is measured for all 16 static hole pairs. It took about two hours to accomplish such a test series. The procedure is repeated for new settings of the wind tunnel speed, thus giving a total number of 9 test series, labeled from A to I.

The experimental results are presented in Table 2 where all the pressure differences measured under varying conditions are given. The pressure difference is measured by the inclined tube micromanometer and is presented in [mm Alcohol × 25]. The table also records the test series A–I with values for the wall shear stress τ_ω (determined from the mean velocity profile) measured 1,745 mm downstream of the tripping strip, i.e. at the same streamwise location as the center of the static hole pairs. Each static hole pair is given its own number, ranging from 1 to 16, as can be seen in Table 2.

Figure 4 is a scrutiny of the pressure readings to see if these exhibit the same ratio from case to case for each static hole pair. It is seen that this is indeed the case, the dotted lines being guidelines to help visualizing this fact. It is noticed that the static hole pair No. 6 (◆) gives consequently higher readings than expected from the results of the others. This reflects influences of details in the manufacturing of the static hole pair. When carefully scrutinizing details of the test plate minute imperfections were found, not in the pressure hole, but in the upstream connection between the acryl plug and the test plate. The

Table 2. Measured static hole pair pressure differences. The values are given in [mm Alcohol × 25]

Pair No.	d [mm]	α	Test series A	B	C	D	E	F	G	H	I
1 ○	10	20°	91.8	6.0	33.0	50.2	16.5	12.2	28.0	151.5	77.5
2 ●	7	20°	80.7	3.7	28.0	42.1	13.0	9.9	23.5	109.0	66.5
3 □	5	20°	64.2	2.7	24.5	36.2	10.8	6.8	17.1	90.5	48.0
4 ■	4	20°	57.7	2.2	19.0	29.7	8.0	5.5	14.2	72.0	45.5
5 ◇	3	20°	44.2	1.0	13.5	21.5	5.0	3.7	10.0	59.5	31.5
6 ◆	2.5	20°	48.7	1.7	13.8	24.6	6.4	3.7	9.0	55.0	28.0
7 △	2.0	20°	33.5		10.0	16.0	5.0	2.7	6.1	44.5	20.5
8 ▲	1.5	20°	26.0		8.6	13.5		2.0	4.4		16.5
9 ▽	1.0	20°	12.9		3.5	6.9	1.0				
10 ○	2.0	50°	4.3		1.5	2.0	0.8				
11 ●	2.0	40°	5.0	0.5	2.0	2.3	1.0				
12 □	2.0	30°	15.5	0.8	5.0	8.0	2.3				
13 ■	2.0	25°	15.0	1.0	5.0	8.0	2.6				
14 ◇	2.0	20°	29.3	0.2	7.8	14.7	3.0				
15 ◆	2.0	15°	42.0	1.2	11.6	21.5	4.6				
16 △	2.0	12.5°	57.7	1.5	18.9	29.5	7.0				
τ_ω [N/m²]			1.067	0.100	0.513	0.739	0.256	0.182	0.404	1.398	0.903

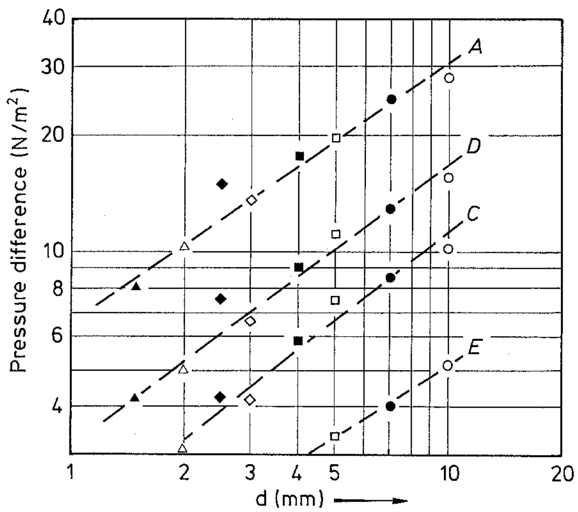

Fig. 4. Readings of static hole pairs No. 1–8 for the cases A, C, D and E

imperfections were in the form of a shallow cavity and were found for both pressure hole pair No. 6 and pair No. 14. The implications and effects of the shallow cavity on the static hole pressure reading were not studied any further. However, the streamwise extent of the cavity is $x^+ = x\,v_*/\nu = 63$ for the test serie A and $x^+ = 20$ for test serie C (test series with highest respectively lowest values for the wall shear stress). Green and Coleman (1973) concluded, based on investigations by Michalke (1972), that the viscous sublayer would be able to traverse a cavity of streamwise extent $x^+ \approx 55$ without significant departure from a purely viscous flow. The present values for x^+ are about of the same magnitude and may thus indicate that the high pressure readings from static hole pair No. 6 are due to upstream disturbances in the viscous sublayer.

Table 3. Coefficients C and n

Pair No.	C	n	Standard derivation
1	0.0244	0.83702	11.0%
2	0.0340	0.79466	8.4%
3	0.0446	0.77011	4.3%
4	0.0550	0.74949	5.3%
5	0.0911	0.66249	8.6%
7	0.0931	0.72400	11.9%
8	0.1256	0.67977	11.7%
9	0.2554	0.55542	1.0%

Figure 5 shows a correlation between the measured static hole pressure differences and the wall shear stress τ_ω. It is noticed that a regular behaviour is present and a calibration relation of the following type is suggested:

$$\tau_\omega = C(\Delta p)^n \tag{5}$$

where C and n are constants depending on the diameter d and the angle α characterizing a static hole pair. The regularity of these results is best appreciated by using a best fit procedure to fit straight lines through the data points of Fig. 5. The result is shown in Table 3 where the constants of Eq. (5) are listed for the static hole pairs No. 1–5 and 7–9. The regularity with which the constants C and n vary with the diameter d is best illustrated in Fig. 6. Static hole pairs No. 5 and 9 are seen to depart somewhat from the general behaviour, but in all cases

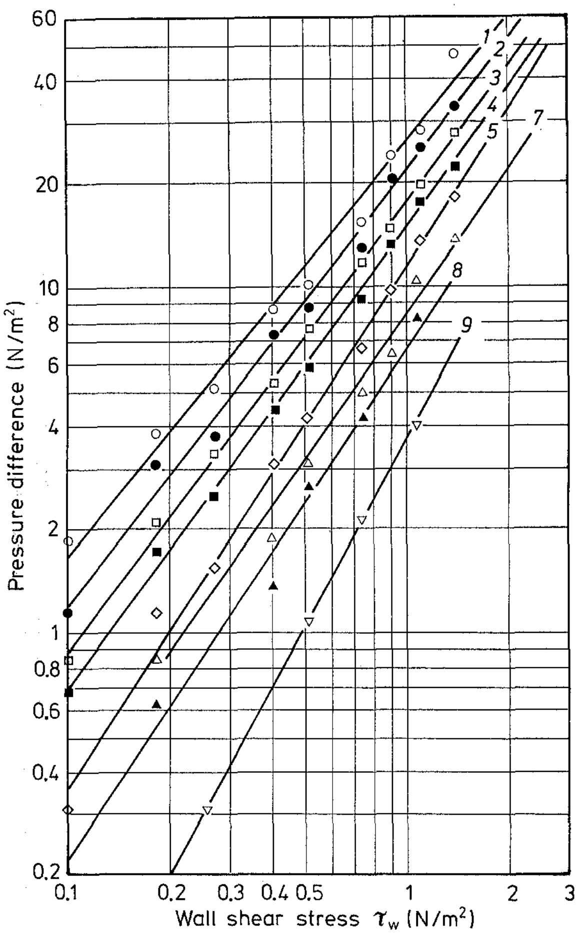

Fig. 5. Measured pressure differences as a function of the wall shear stress τ_ω. Static hole pairs No. 1–7 and 8–9

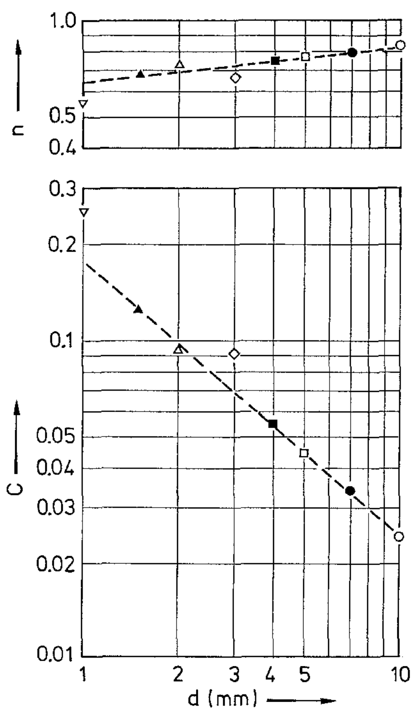

Fig. 6. Variation of the parameters C and n as a function of the diameter d

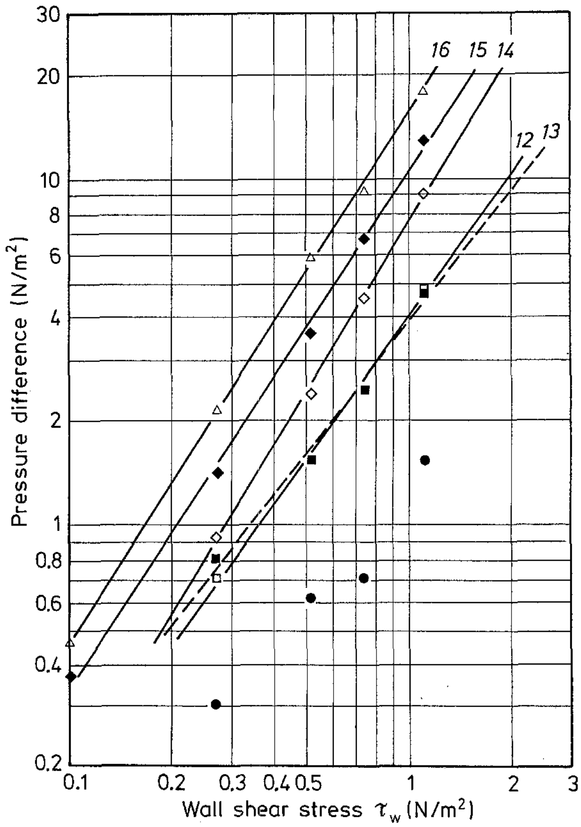

Fig. 7. Measured pressure differences as a function of the wall shear stress τ_ω. Static hole pairs No. 11–16

Eq. (5) seems to apply with a satisfactory degree of accuracy. Even though the static hole pair No. 6 gives readings which fall somewhat outside the pattern demonstrated above, it still exhibits results which obey the calibration curve (5).

The next step is to investigate the influence of a change in the angle α of the static hole pair. This is done through the readings of the static hole pairs No. 10–16. The first one (No. 10) is rejected because the readings are too small to be significant. The other readings are exhibited in Fig. 7 and the main conclusion to be drawn from these is that angles greater than 30° seem not to be recommendable. It is noticed that no regular behaviour seems to be traceable except for the fact that smaller angles give larger pressure differences and thus greater sensitivity. It is also worth mentioning that a certain lower limit for α seems to be set by practical difficulties in manufacturing the pressure hole.

5 Conclusions

The experiments are conducted to investigate the possibility to utilize the difference in pressure readings from a static hole pair for determination of the local wall shear stress. The number of flow variables that could influence the static hole pressure readings were reduced as far as possible, i.e. a two-dimensional zero pressure gradient mean flow. The calibration values for the local wall shear stress were determined from the universal scaling laws for the inner part of the turbulent boundary layer which were measured by using hot-wire anemometry.

By varying the diameter of the second hole in a static hole pair it is shown that the results for the pressure difference correlate well with the local wall shear stress. The diameter variation may be incorporated in the suggested calibration correlation. However, the investigations aimed at obtaining a similar incorporation of variations in the angle of the second hole axis w.r.t. the plane surface were not successful.

Although it is shown that the static hole pair is a device which can favourably be used to measure the local wall shear stress, it should be noted that any static hole pair to be used must be calibrated separately and that the C- and n-values quoted in Table 3 apply only to the static hole pairs tested. C- and n-values must be experimentally established in each case, even though the values given may be used as guiding values.

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